Problems

Ted Eisenberg, Section Editor

****

This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at <http://www.ssma.org/publications>.

Solutions to the problems stated in this issue should be posted before October 15, 2015

- **5355:** *Proposed by Kenneth Korbin, New York, NY*

  Find the area of the convex cyclic pentagon with sides

  $$(13,\ 13,\ 12\sqrt{3}+5,\ 20\sqrt{3},\ 12\sqrt{3}-5).$$

- **5356:** *Proposed by Kenneth Korbin, New York, NY*

  For every prime number p there is a circle with diameter $4p^4+1$. In each of these circles, it is possible to inscribe a triangle with integer length sides and with area

  $$(8p^3)(p+1)(p-1)(2p^2-1).$$

  Find the sides of the triangles if $p=2$ and if $p=3$.

- **5357:** *Proposed by Neculai Stanciu, "George Emil Palade" School, Buzău, Romania and Titu Zvonaru, Comănesti, Romania*

  Determine all triangles whose side-lengths are positive integers (of which at least one is a prime number), whose semiperimeter is a positive integer, and whose area is equal to its perimeter.

- **5358:** *Proposed by Arkady Alt, San Jose, CA*

  Prove the identity $\displaystyle\sum_{k=1}^{m} k\binom{m+1}{k+1}r^{k+1} = (r+1)^m(mr-1)+1.$

5359: *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain.*

Let a, b, c be positive real numbers. Prove that

$$\sqrt[4]{15a^3b+1}+\sqrt[4]{15b^3c+1}+\sqrt[4]{15c^3a+1} \le \frac{63}{32}(a+b+c)+\frac{1}{32}\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right).$$